See 1.6 Notes: Complex numbers

$$a + bi$$

real # imaginary #

Reminders:

a. $\sqrt{-1} = i \rightarrow \quad \textit{therefore} \quad i^2 = -1$

b. $\sqrt{-9} = \sqrt{9} \bullet \sqrt{-1} = 3i$

c. $(3i)^2 = 3^2 \bullet i^2 = 9(-1) = -9$

$$d. \quad (3+i)^2 = (3+i)(3+i)$$

$$= 9 + 6i + i^2$$

$$= 9 + 6i + -1$$

$$= 8 + 6i$$

e. Simplify using the conjugate:

$$\frac{i}{2+4i} = \frac{i\,(2-4i)}{(2+4i)(2-4i)}$$

$$= \frac{2i-4i^2}{4-16i^2}$$

$a+bi$ form

$$= \frac{2i+4}{4+16} = \frac{4+2i}{20} = \frac{1}{5}+\frac{1}{10}i$$

Reminder: $i^2 = -1$

Reminder from ch.5 notes:

Principal values determine specific solutions rather than an infinite number of solutions.

$$\left.\begin{array}{l}\sin x\\ \tan x\end{array}\right\} -90^\circ \le x \le 90^\circ \qquad Quadrants\ I, IV$$

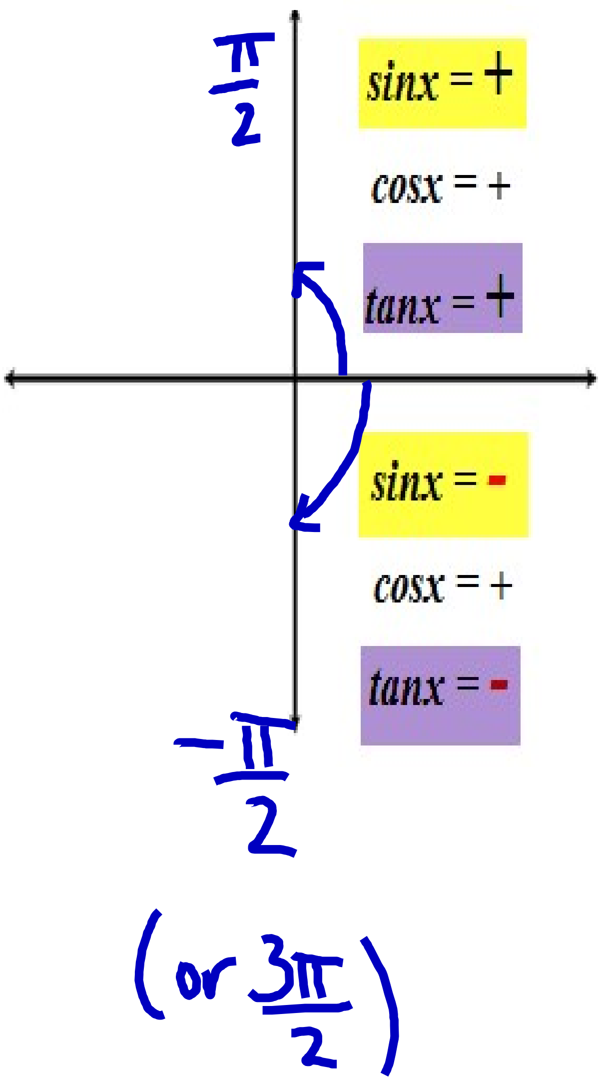
$-\pi/2 \qquad \pi/2$

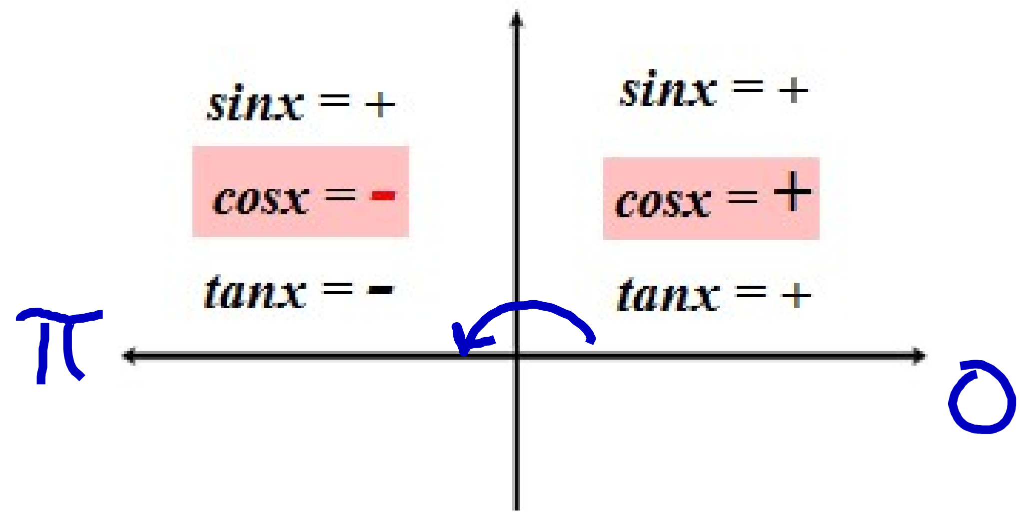
$$\cos x \} \quad 0^\circ \le x \le 180^\circ \qquad Quadrants\ I, II$$

$0 \qquad \pi$

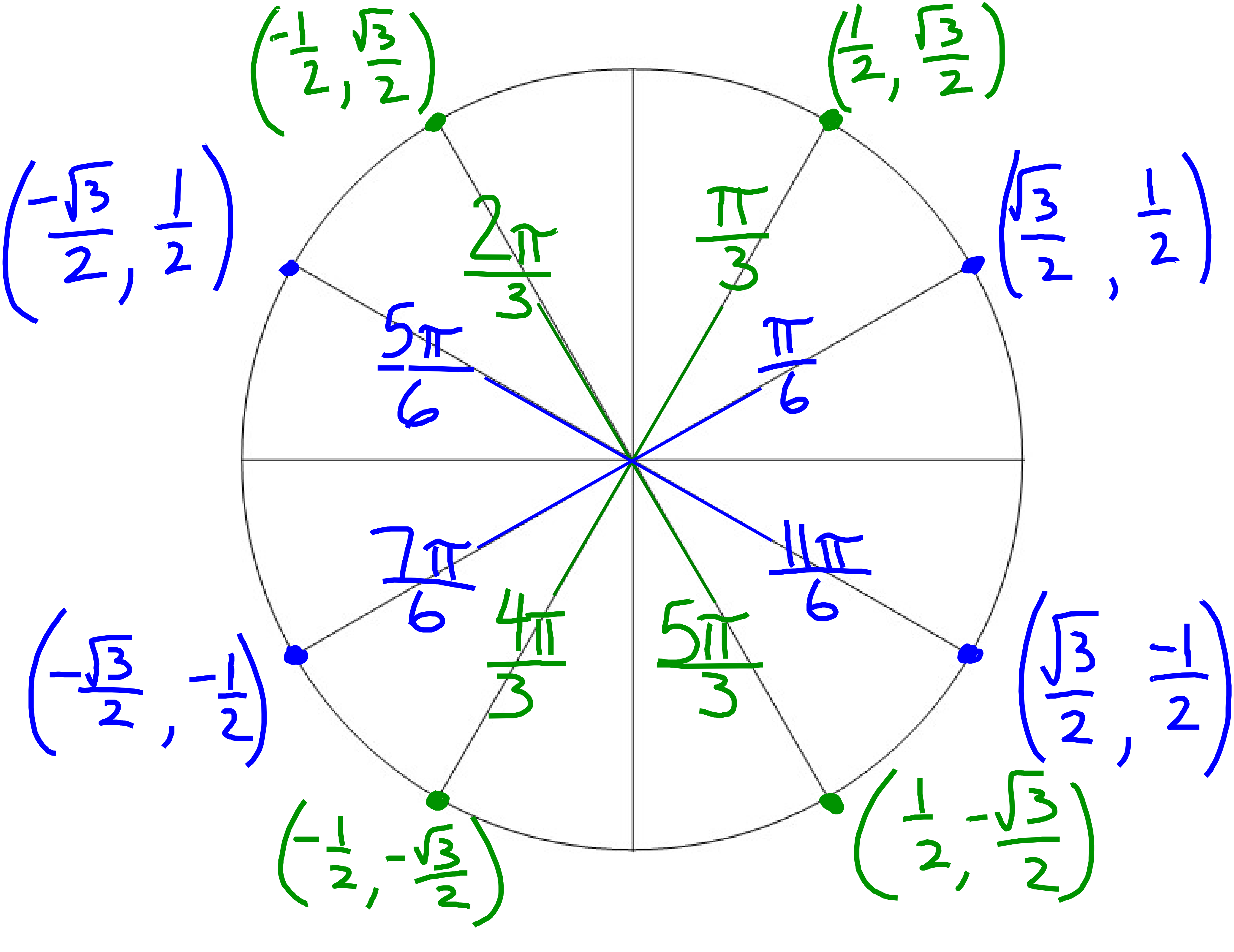
$$\left.\begin{matrix}\sin x \\ \tan x\end{matrix}\right| -\frac{\pi}{2} \le x \le \frac{\pi}{2} \quad \textit{Quadrants I, IV}$$

There is only one set of positive and negative values for the given quadrants.

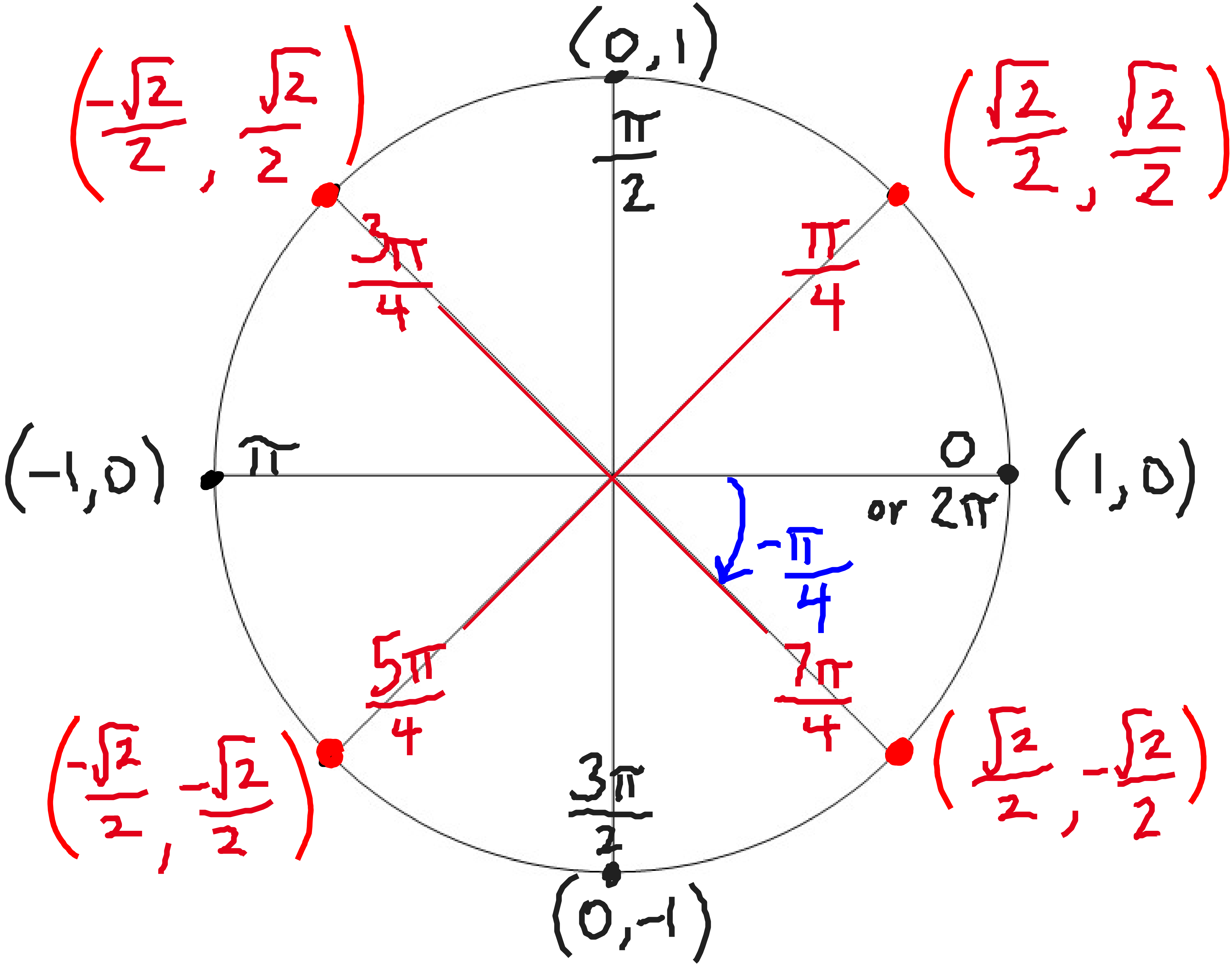




$$\cos x \} \quad 0° \le x \le \pi \quad \textit{Quadrants I, II}$$



$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

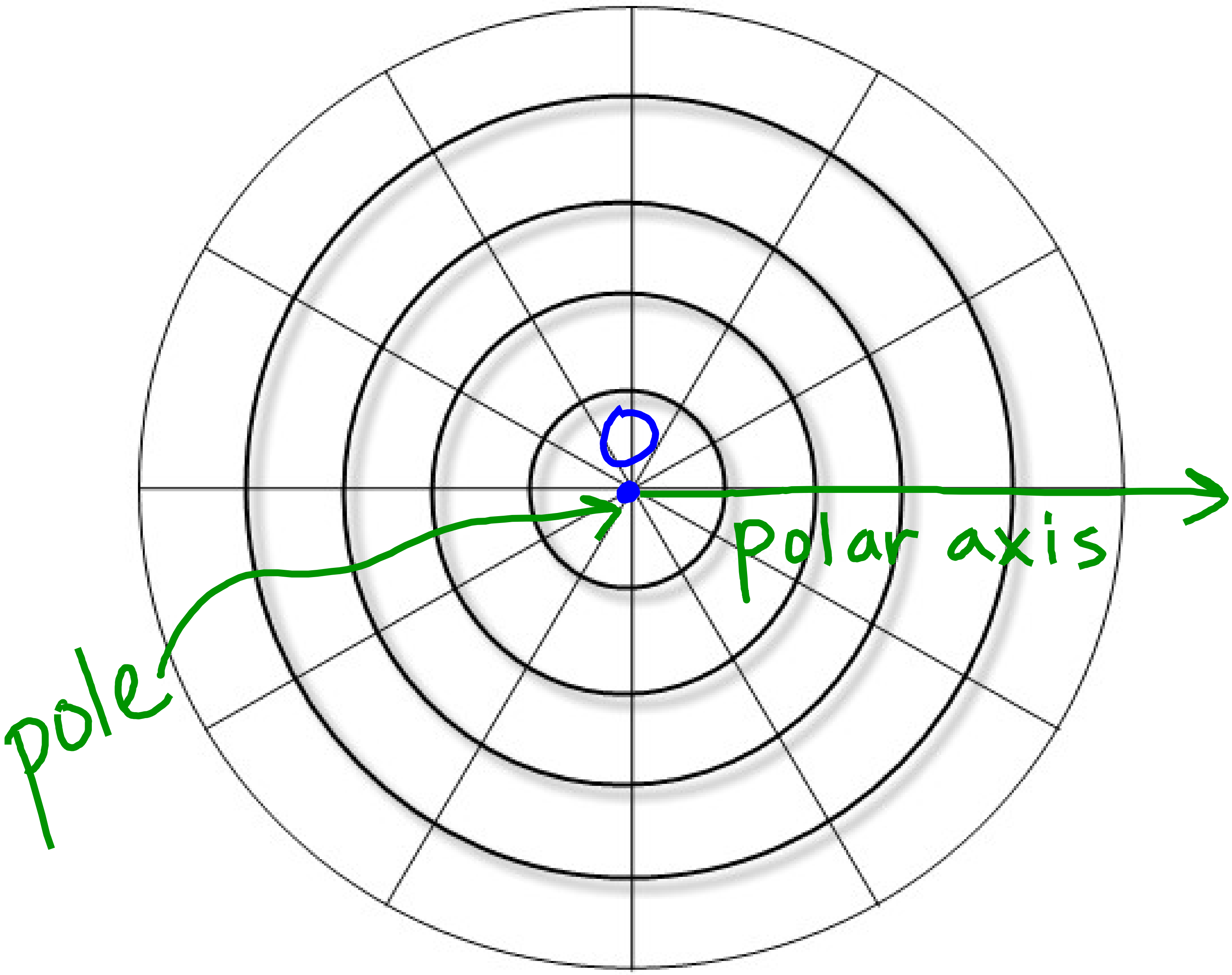

(0, 1)
π/2
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
3π/4
π/4
(−1, 0)
π
0
or 2π
(1, 0)
−π/4
5π/4
7π/4
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
3π/2
(0, −1)

TODAY! Notes 8.1 Polar Coordinates

Polar Coordinates: used to record the position of an object using a fixed point and an angle made with a fixed ray.

Pole: the fixed point "O" at the origin

Polar Axis: the horizontal ray directed toward the right from the pole.


O
polar axis
pole

Notes 8.1 continued…

(r, θ): the polar coordinates that locate point P

r = distance from point O to point P

θ = angle between polar axis and the ray $\overrightarrow{OP}$

negative

NOTE: if r < 0, locate the terminal side, then move in the opposite direction

(move away from given angle)

Conversion from Polar Coordinates to Rectangular Coordinates

$(r, \theta) \rightarrow (x, y)$

polar rectangular

$$x = r\cos\theta, \quad y = r\sin\theta$$

Conversion from Rectangular Coordinates to Polar Coordinates

$(x, y) \rightarrow (r, \theta)$

rectangular polar

$$r = \sqrt{x^2 + y^2} \quad or \quad r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x} \quad (x \neq 0)$$

WARM-UP ON TODAY'S RESOURCE PAGE:

Ch.8 Honors Trig/Precalc **NAME:** **PER:**

8.1 #5-10, 12,14, 17-22, 25-28 AND unit circle, complex numbers#1-5

Warm-up:

M = (,)

A = (,)

T = (,)

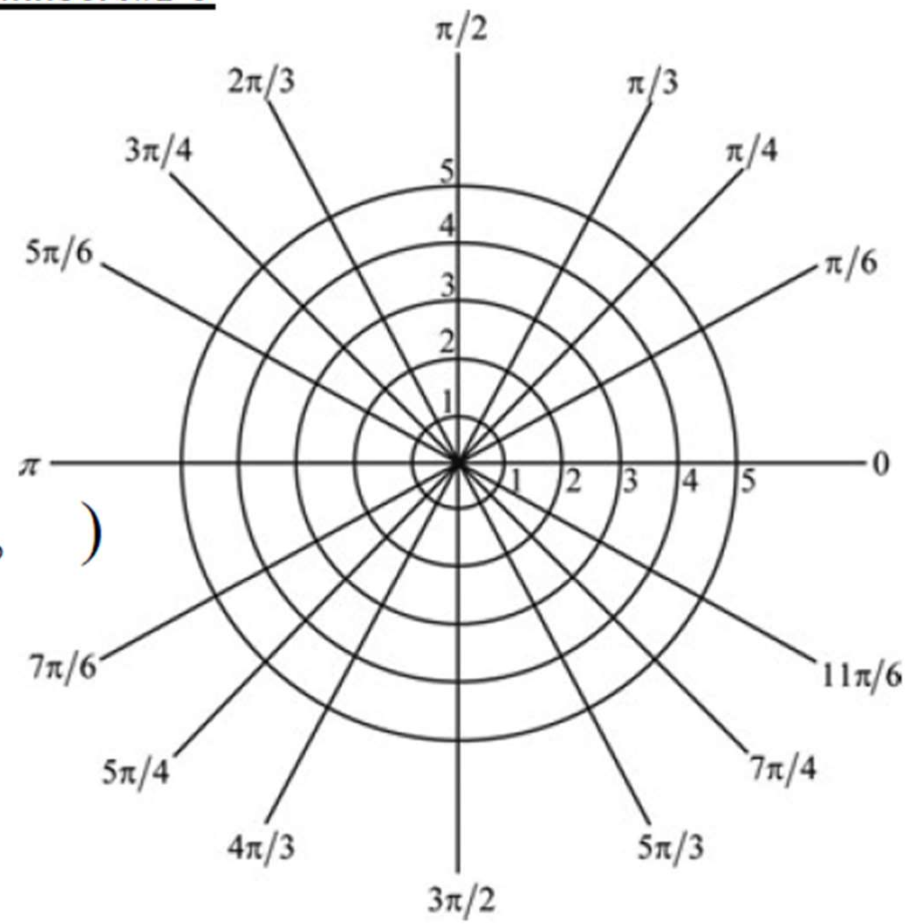
H = (,)

F is similar to → #12 and #14

F = (,) (,) (,) (,)

U = (,)

N = (,)



$M = (3 , 60°)$

$A = (4 , \frac{-5\pi}{6})$

$T = (-2 , \frac{5\pi}{3})$

$H = (-5 , \frac{-\pi}{6})$

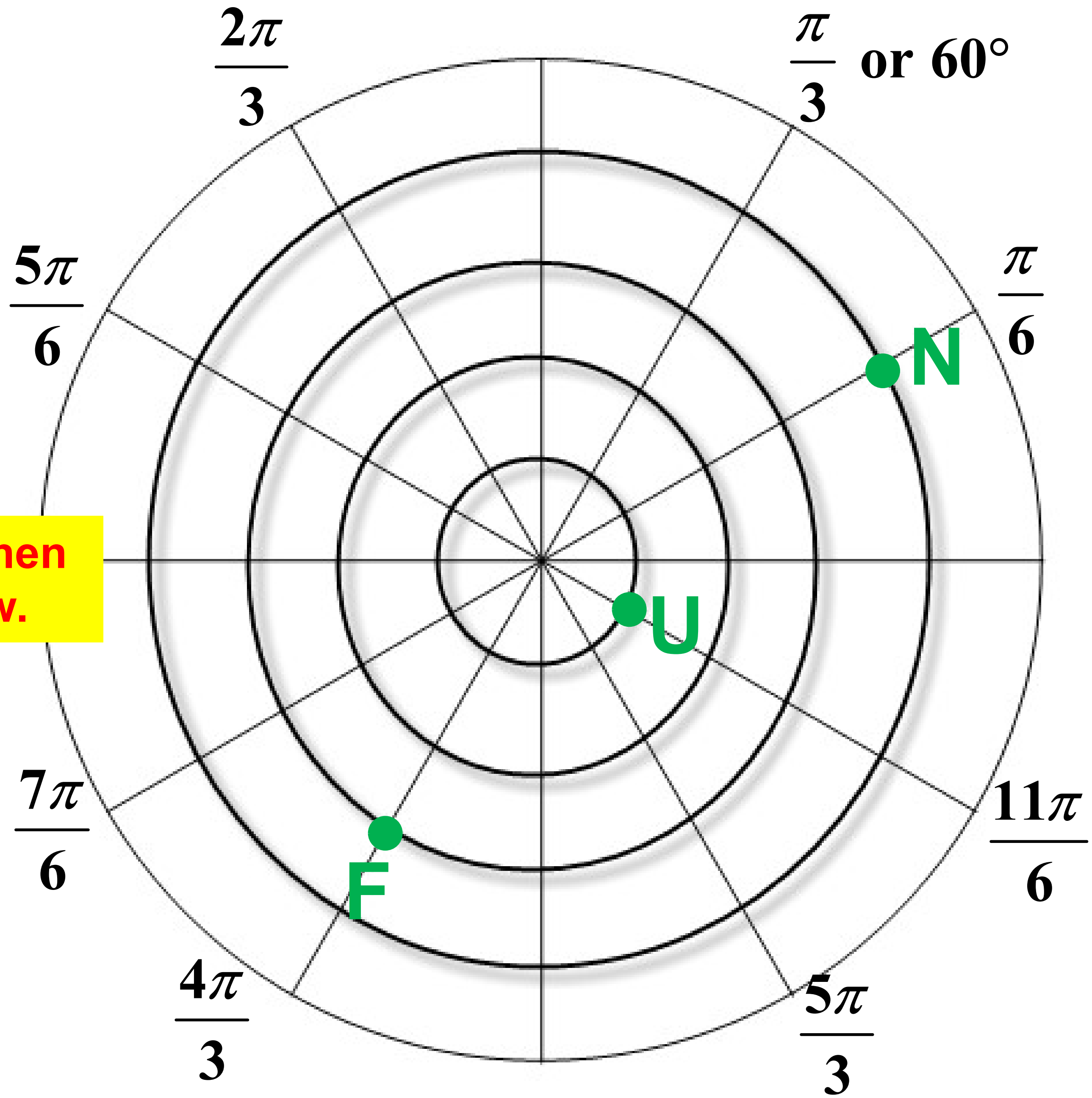
Plot the points given above, then name the points listed below.

$F = (\quad , \quad)$

Name F four different ways!

$U = (\quad , \quad)$

$N = (\quad , \quad)$



$M = (3, 60°)$

$A = (4, \frac{-5\pi}{6})$

$T = (-2, \frac{5\pi}{3})$

$H = (-5, \frac{-\pi}{6})$

or $F=(3, \frac{-2\pi}{3})$

or $F=(-3, \frac{\pi}{3})$

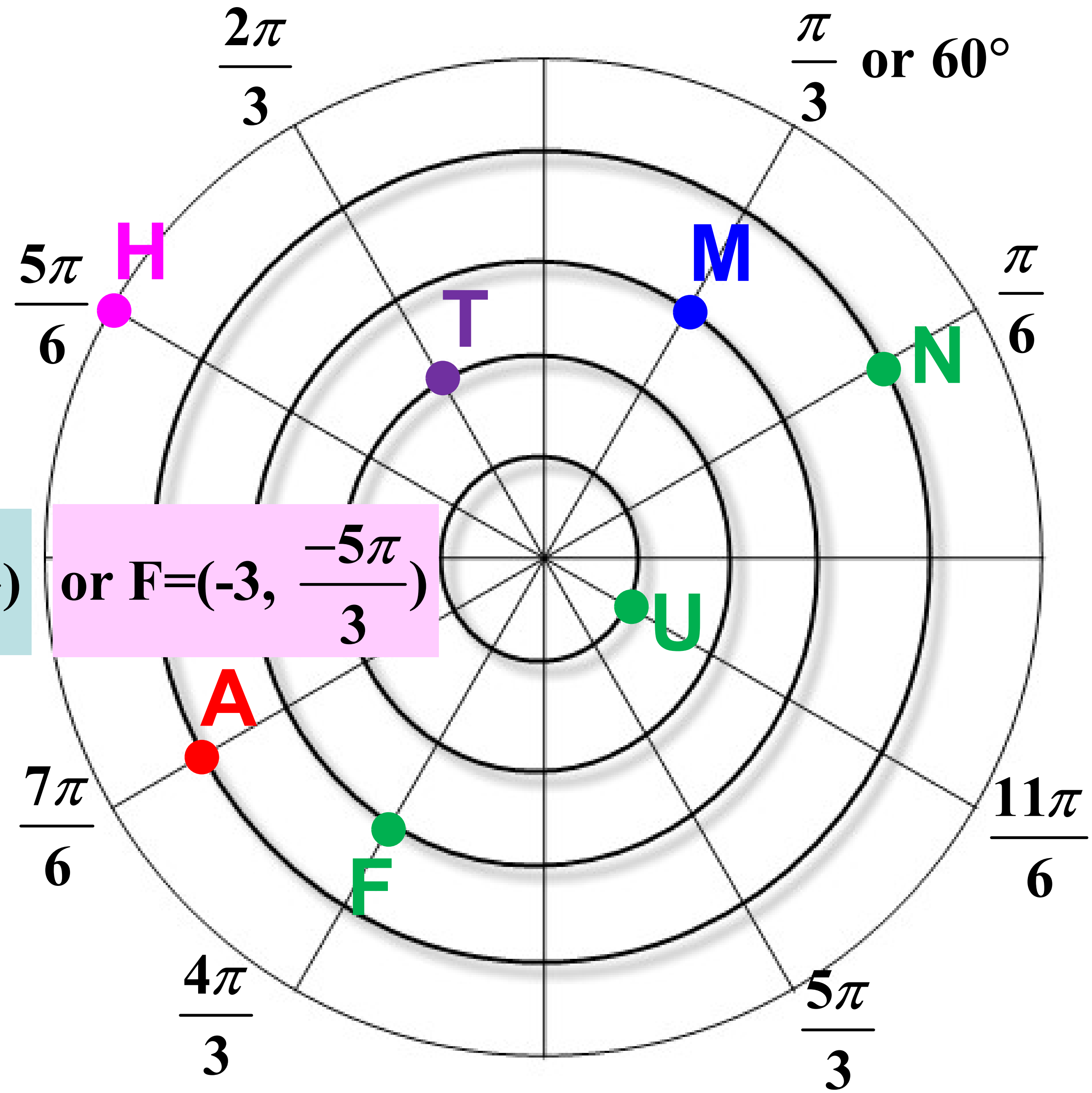
or $F=(-3, \frac{-5\pi}{3})$

$F = (3, \frac{4\pi}{3})$

Name F four different ways!

$U = (1, \frac{11\pi}{6})$

$N = (4, \frac{\pi}{6})$

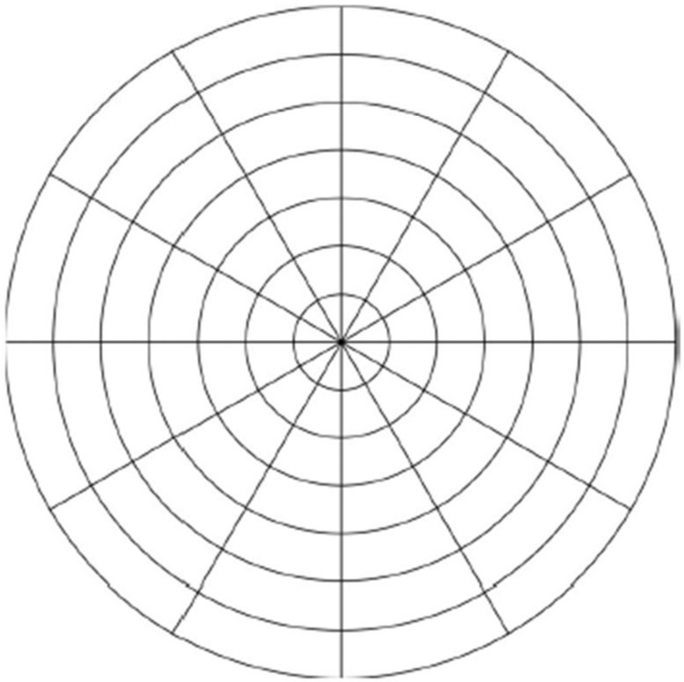


Unless stated otherwise, the default is to use $r > 0$ and $0 \leq \theta < 2\pi$ (both positive values!)

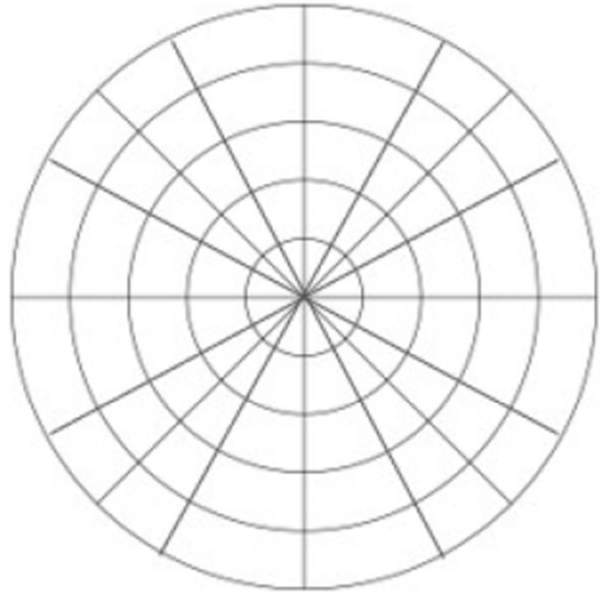
PLEASE FOLLOW THE INSTRUCTIONS ON TODAY'S RESOURCE PAGE:

8.1 #5-10, 12,14

#5-8 graph, label coordinates next to each point

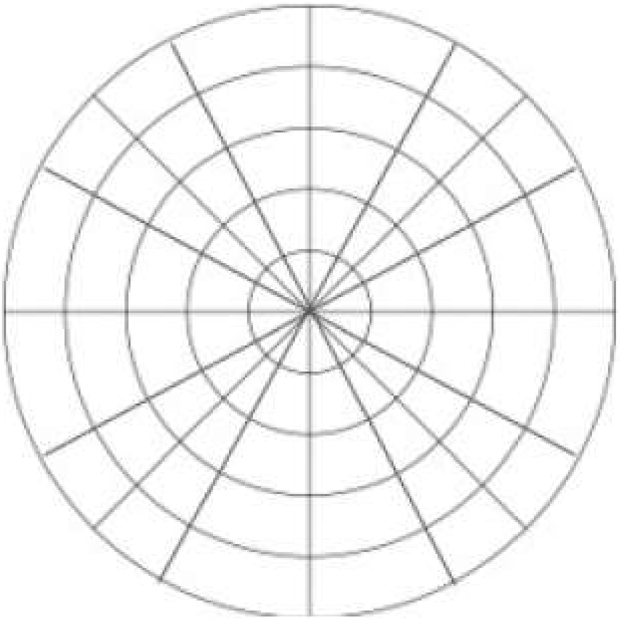


#9-10 graph, label coordinates next to each point



SOME OF THE BOOK INSTRUCTIONS HAVE BEEN SLIGHTLY MODIFIED:

#12 plot point, label given coordinates, then list **three** other possible coordinates for the same point where $-2\pi \leq \theta \leq 2\pi$

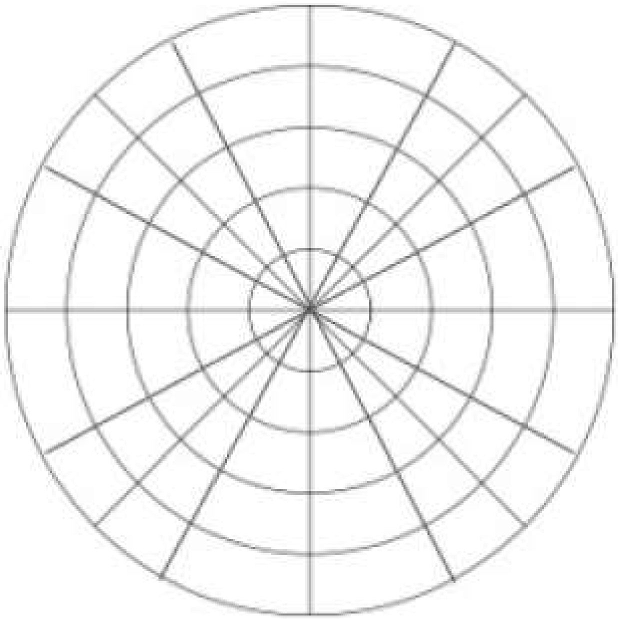


(,)

(,)

(,)

#14 plot point, label given coordinates, then list **three** other possible coordinates for the same point where $-2\pi \leq \theta \leq 2\pi$



(,)

(,)

(,)

over →

PLEASE FOLLOW THE INSTRUCTIONS AND ALSO COMPLETE THE REVIEW PROBLEMS:

8.1 #17-22: write given coordinates, then identify point

CHECK EVEN BOOK ANSWERS:
(#12,14,18,20,22,26,28)

$\left(2,\frac{2\pi}{3}\right)$ $\left(-2,\frac{5\pi}{3}\right)$ $\left(3,\frac{3\pi}{2}\right)$ $\left(-\sqrt{3},1\right)$

$\left(2,-\frac{4\pi}{3}\right)$ $\left(2,-\frac{5\pi}{4}\right)$ $\left(-2,-\frac{\pi}{4}\right)$ $\left(-2,\frac{7\pi}{4}\right)$

P Q R

8.1 #25-28 show all steps on a separate sheet of paper!

reminder: $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$

no calculator, refer to unit circle to solve

Review of Unit Circle and Complex Numbers (see notes 1.6)

1. Complex numbers ***(show work on a separate sheet of paper!)***

A. $(2 - 3i)(7 - 4i)$

B. $(1 + 4i)^2$

C. $(2 - 3i) + (7 - 4i)$

D. $(2 - 3i) - (7 - 4i)$

E. $\frac{2+i}{1+2i}$ (hint: use conjugate)

F. $\frac{3-2i}{-4-i}$ (hint: use conjugate)

CHECK ANSWERS#1, 3-5

I I II IV $\frac{\sqrt{3}}{3}$ $-\frac{\sqrt{3}}{2}$

$\frac{y}{x}$ $\frac{x}{y}$ $\frac{1}{x}$ $\frac{1}{y}$ x y

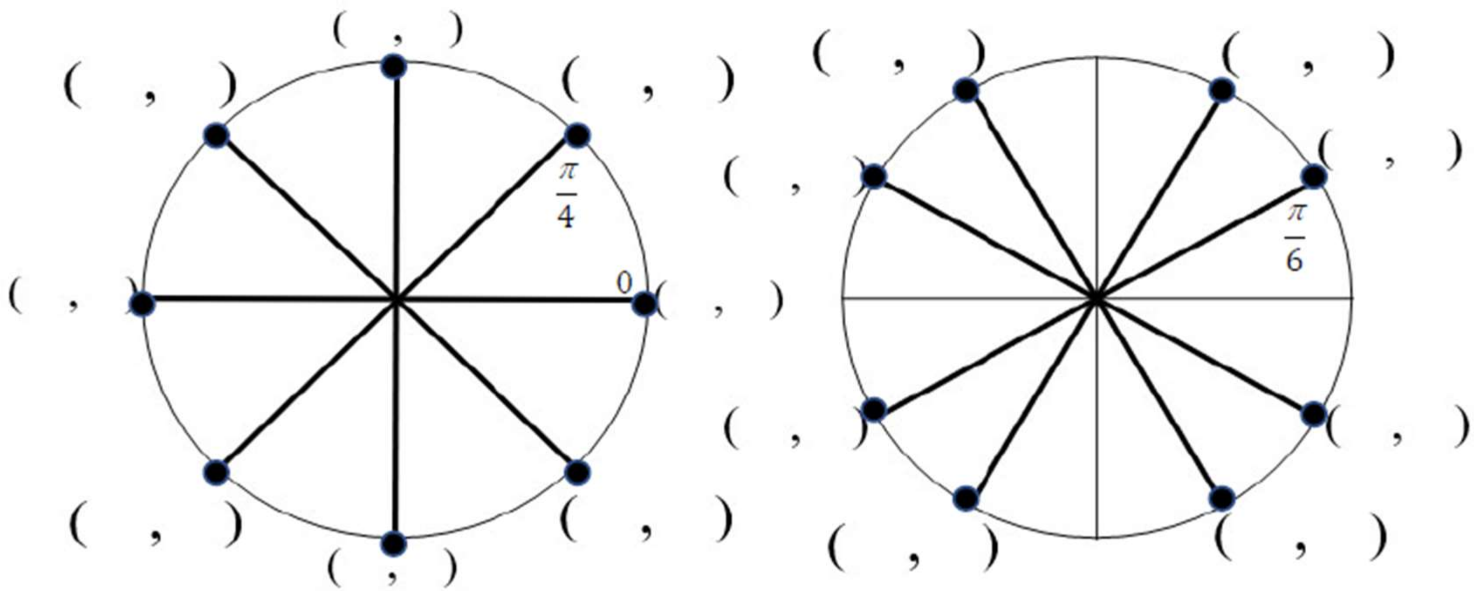
$\frac{1}{2}$ $\frac{\pi}{2}$ $\frac{\pi}{4}$ $\frac{7\pi}{4}$ $\sqrt{3}$

$-15 + 8i$ $-5 + i$ $-\frac{10}{17}+\frac{11}{17}i$

$\frac{4}{5}-\frac{3}{5}i$ $2 - 29i$ $9 - 7i$

ALSO COMPLETE THE REVIEW PROBLEMS:

2. Label all radian values AND coordinates of each given terminal point.
(You will need to have this information memorized again for the ch.8 test!)



3. Define each function in terms of x and y (based on the unit circle with r = 1.)

$\sin\theta =$ $\quad$ $\csc\theta =$ $\quad$ $\cos\theta =$ $\quad$ $\sec\theta =$ $\quad$ $\tan\theta =$ $\quad$ $\cot\theta =$

4. Principal Values (see notes from chapter 5.)

To find a ***unique*** solution for Sinθ and Tanθ, refer only to Quadrant ___ or ___.

To find a ***unique*** solution for Cosθ, refer only to Quadrant ___ or ___.

5. Evaluate using the unit circle. Use principal values when finding the inverse, $0 \le \theta < 2\pi$. <u>No calculator!</u>

A. $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$ $\quad$ B. Arctan(1) $\quad$ C. $\text{Cos}^{-1} 0$ $\quad$ D. $\sin\left(\frac{13\pi}{6}\right)$

hint: rewrite as $\text{Sin}\theta = -\frac{\sqrt{2}}{2}$

E. $\cot\left(-\frac{5\pi}{3}\right)$ $\quad$ F. $\sin[\text{Arctan}(-\sqrt{3}\ )]$ $\quad$ G. $\cot(\text{Cos}^{-1}(-1) + \text{Sin}^{-1}\frac{1}{2})$

Show all steps for F and G

ALSO COMPLETE THE REVIEW PROBLEMS:

3. Define each function in terms of x and y (based on the unit circle with r = 1.)

$\sin\theta =$ $\csc\theta =$ $\cos\theta =$ $\sec\theta =$ $\tan\theta =$ $\cot\theta =$

4. Principal Values (see notes from chapter 5.)

To find a ***unique*** solution for Sinθ and Tanθ, refer only to Quadrant ___ or ___.

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5. Evaluate using the unit circle. Use principal values when finding the inverse, $0 \le \theta < 2\pi$. <u>No calculator!</u>

A. $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$

hint: rewrite as $\text{Sin}\theta = -\frac{\sqrt{2}}{2}$

B. $\text{Arctan}(1)$

C. $\text{Cos}^{-1}0$

D. $\sin\left(\frac{13\pi}{6}\right)$

E. $\cot\left(-\frac{5\pi}{3}\right)$

F. $\sin[\text{Arctan}(-\sqrt{3})]$

Show all steps for F and G

G. $\cot(\text{Cos}^{-1}(-1) + \text{Sin}^{-1}\frac{1}{2})$